ETH zürich

From evolving to temporal networks: the impact on spreading

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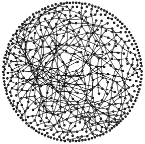
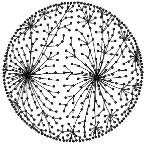
Overview

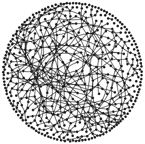
Introduction

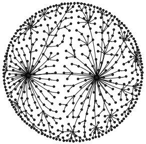




- Model for Speciation Process [Yule, *Philos. Trans. Roy. Soc. Lond. B* (1924)]
- The interaction patterns between agents determine their properties [Bavelas, *Human Organization* (1948)]
- Small-world effect [Milgram, *Sociometry* (1969)]
- Scale-free topologies

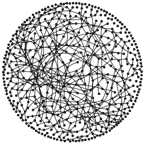
Introduction

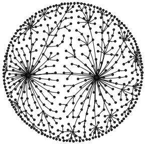




- Models of growth and configurational models
- Preferential attachment model [Barabási, Albert, *Science* (1999)]:
  - One node is added at each time step $N = t$
  - It forms links to m existing nodes. Existing nodes are selected with a probability proportional to their degree
- Ergodic properties
- They reach a stationary state in some of their properties

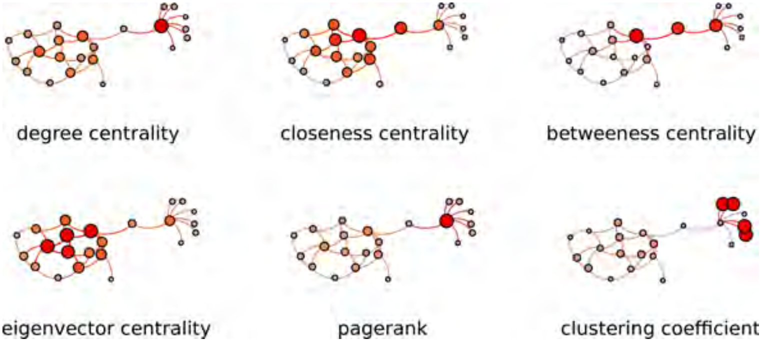
Introduction





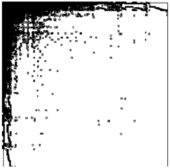
- A Graph $\mathcal{G}(N, E)$, defined by a set of nodes N and of edges E
- Adjacency matrix $a_{ij} = 1$ if $(i, j) \in E$; $a_{ij} = 0$ if $(i, j) \notin E$
- Degree of a node k_i: the number of neighbours
- Different topological properties: *centrality*

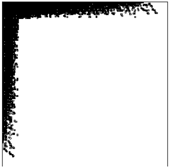
Centrality measures in networks



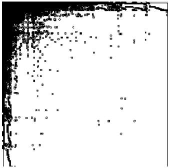
- In different contexts, the importance of an agent in a network is measured by her centrality [Bavelas, *Human Organization* (1948)], [Wasserman, **Social Network Analysis** (1994)]

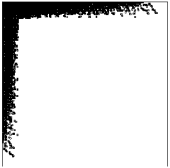
Nestedness in networks





Nestedness in networks





- The neighbourhood of large degree nodes contain the neighbourhood of lower degree nodes
- Typical of highly hierarchical structures
- Core-periphery structure

[König, Tessone, Zenou, *Theoretical Economics* (2014)]

Motivation: Economics

- The Fedwire bank network [Soramaki, *Physica A* (2007)] are nested in the sense that their organisation is strongly hierarchical
- Banks seek relationships with each other that are mutually beneficial
- As a result, small banks interact with large banks for security, lower liquidity risk and lower servicing costs
- large banks may interact with small banks in part because they can extract a higher premium for services and can accommodate more risk

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- Centrality is an indirect measure of link of Bank performance/size [Akram, Christophersen, *working paper* (2011)].

Motivation: Economics

- The network formation process can be viewed as a two-stage game on two separate time scales
- On the fast time scale, all agents simultaneously choose their effort level in a fixed network structure [Ballester, *Econometrica* (2006)]

Individual payoff

$$\pi_i(\mathbf{x}, G, \lambda) \equiv x_i - \frac{1}{2}x_i^2 + \lambda \sum_{j=1}^{n} a_{ij}x_i x_j, \tag{1}$$

In equilibrium, payoff is a function of the agent's centrality in the network

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On the slow time scale, agents receive linking opportunities at a rate α

$$b_i^{\zeta}(j|G(t)) \equiv \mathbb{P}\left(\pi_i^*(G(t) \oplus (i,j), \lambda) + \varepsilon_{ij}\right)$$
$$= \frac{e^{\pi_i^*(G(t)\oplus(i,j),\lambda)/\zeta}}{\sum_{k\in\mathcal{N}\setminus(\mathcal{N}_i\cup\{i\})} e^{\pi_i^*(G(t)\oplus(i,k),\lambda)/\zeta}}.$$

random utility model

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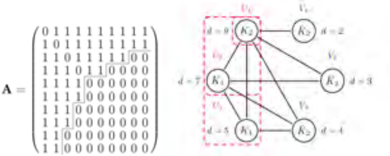
Model Intuition

- If a link has to be created: the best strategy for an agent would be to select the node increases her centrality the most
- If a link has to be deleted: the best strategy for an agent would be to select the node that decreases her centrality the least

Model: Network evolution

- This process generates, at every time step, a *nested network*
- We have shown that the most efficient strategy is independent of the type of centrality agents want to maximise: closeness, betweenness and Bonacich centrality
  - The link must be created to the node with the largest degree the agent is not still connected to
  - For removal, delete the link to the node with the lowest degree



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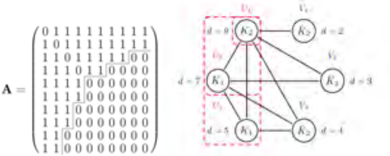
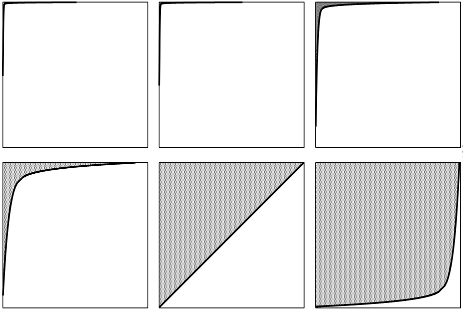
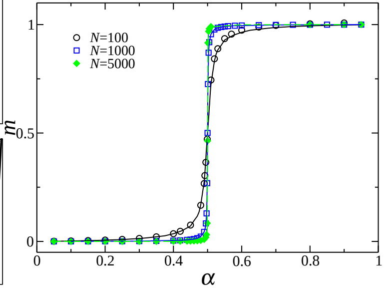
- **Self-reinforcing structure: once established, it is better for the agents to maintain it.** Symmetric choice



Adjacency matrix and network density



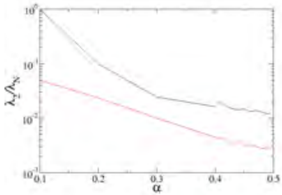


(left) Computer simulations and analytical result for $N = 10^3$, for $\alpha = 0.40, 0.42, 0.48, 0.495, 0.50, 0.505$. (right) Network density *m* as a function of α

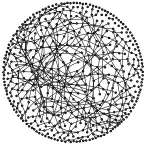
- Large volatility (low α), hierarchical, centralised network
- Low volatility (large α), highly decentralised network
- Sharp transition from hierarchical to flat structures by decreasing volatility

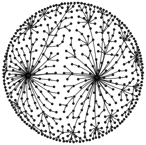
Spreading processes

- Spreading processes: SIR-like models exhibit a threshold in infection rate above which spreading reaches all the system
- In networks with scale-free distribution, the threshold is 0. Thereby all outbreaks cover the complete system
- when periodic dynamics is considered, the spreading depends on topological properties of the system determines the synchronisability. Given the Laplacian Matrix, λ_N/λ_2: the smaller, the easier to get collective phenomena by coupling



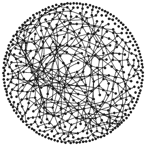
The role of network structure

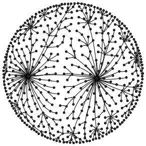




- Single-scale networks (random, small-world), show a transition towards synchronisation similar to those in mean-field (effects of clustering, average-path length)
- networks with a *scale-free* –$\gamma \leq 3$– degree distribution show a different kind of transition (at very low –zero?– coupling strength)
- The nature of the phase transition may change (from second to first order) with different coupling schemes...

Evolving networks: conclusions





- When thinking of network evolution, usual modelling approaches consider either
  - ties between nodes to be persistent
  - links have a lifetime long enough such that the backbone of the network builds-up

Temporal networks: Introduction

- This neglects the fact that the interacting units might have limited capabilities
  - capacity constrains
  - cognitive capabilities
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We want to address

How does the network evolution affect the global dynamical properties of a system?

e-MID overnight market

Overnight market

- An unsecured (electronic) market for interbank deposits
- Market participants can choose their counterparties

Partnering for Liquidity
e-MID

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- [Iori *et al.*, *JEDC* (2008)], Finger, Fricke, Lux KWP_17822012, [Hatzopoulos, Iori , *DoE City Univ. London 12/04* (2012)]

Data

- We consider all the ON transactions in the system. They account for 95 % of all transactions
- Every day, we create a directed network with Banks representing nodes n_i^t. Edges $e_j^t = (n_1^t, n_2^t)$
- All the transactions from 01.01.1999 to 20.09.2012: 5016 network snapshots

[Tessone, Quattrochini ,Caldarelli, *working paper* (2014)]

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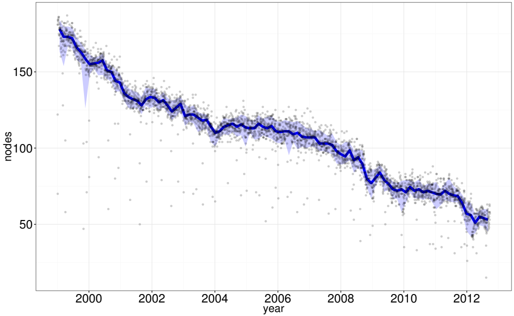
- All the transactions from 01.01.1999 to 20.09.2012: 5016 network snapshots
- Aggregated network: $N_e = 350$, $\langle k \rangle = 80.55$,

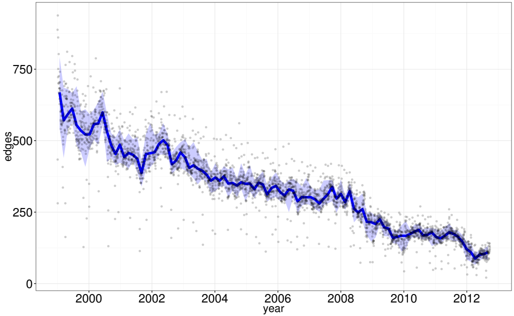
[Tessone, Quattrochini ,Caldarelli, *working paper* (2014)]

Empirical analysis



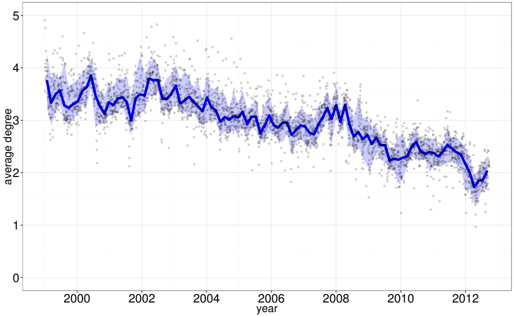
Total number of **nodes**. Daily network as a function of time. Maturity: ON

Empirical analysis



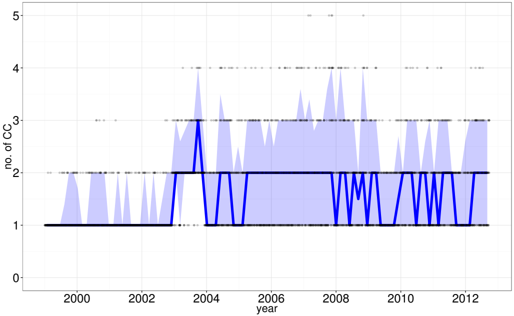
Number of **edges**. Daily network as a function of time. Maturity: ON

Empirical analysis



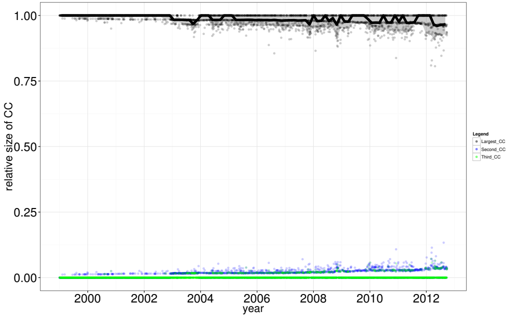
Average degree. Daily network as a function of time. Maturity: ON

Empirical analysis



Number of connected components in the network. Maturity, ON

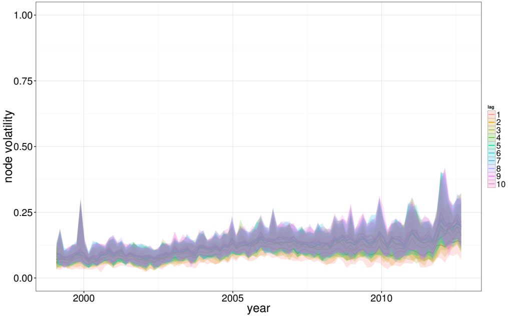
Empirical analysis



Relative size of connected components the network every day as a function of time. Maturity, ON

Network volatility

Proportion of (nodes or edges) which are present in the network at time *t*, but not after *d* days.
In the case of considering $d = 1$, it is the proportion of nodes or edges that dissapear after one day



Volatility of nodes in the network every day as a function of time. Maturity, ON, for different lags.

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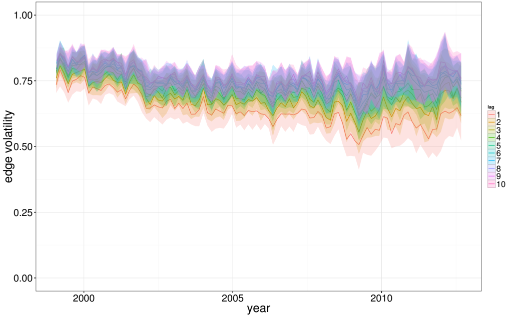
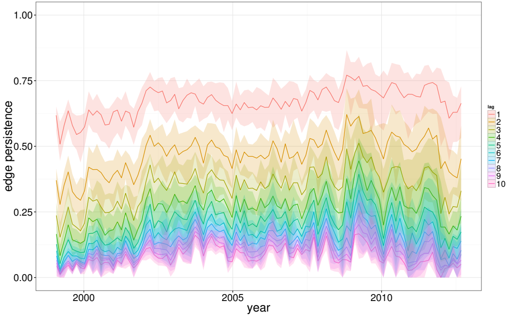


Volatility of edges in the network every day as a function of time. Maturity, ON, for different lags.

Graph persistence

Proportion of (nodes or edges) which are present in the network at time t, and that keep being present after d days.
In the case of considering $d = 1$, it is the proportion of nodes or edges that remain in the network



Node persistence as a function of time. Maturity, ON, for different lags.

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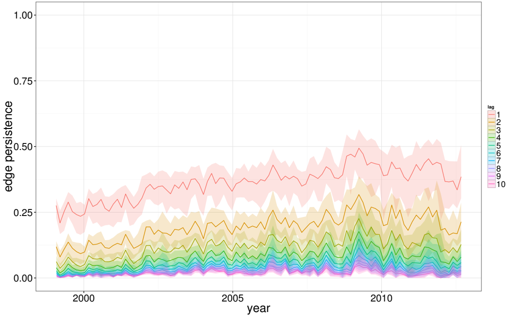
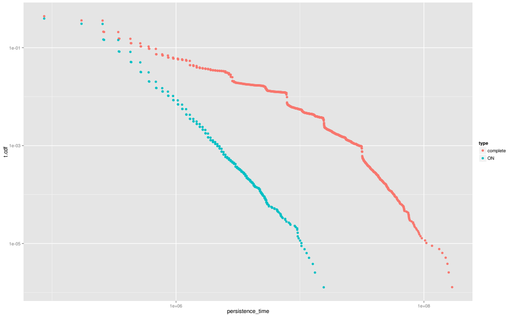


Edge persistence as a function of time. Maturity, ON, for different lags.

Lifetime of links

Lifetime of edges:



Distribution of lifetimes of edges in the network Maturity, ON, all.

Next steps

- Understand network dynamics in this system, to model them
- Network volatility and *distress* propagation: How does network volatility increase distress propagation in this network? To what extent?

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- Understand network dynamics in this system, to model them
- Network volatility and *distress* propagation: How does network volatility increase distress propagation in this network? To what extent?
- Develop a stress test that does not not make any assumptions on the balance sheet structure, and takes into account simply the time evolution of the network.
- Two complementary ingredients:
  - *Temporal* topological features of the network
  - Dynamical processes on rapidly evolving networks

Is it safe to aggregate temporal networks?

It is common practice, **BUT...**

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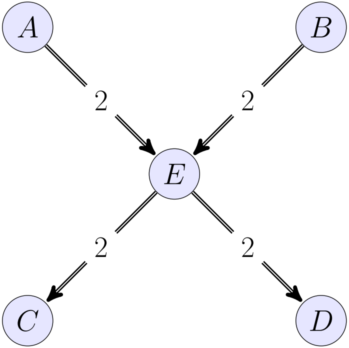


What can we learn from this picture?

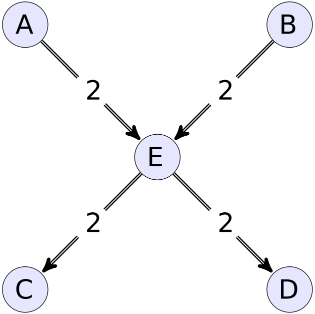
Is it safe to aggregate temporal networks?

It is common practice, **BUT...**

sources

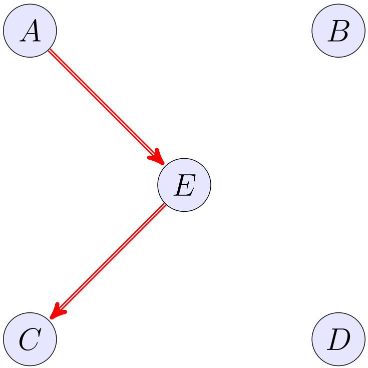
information, infection, etc..

destinations



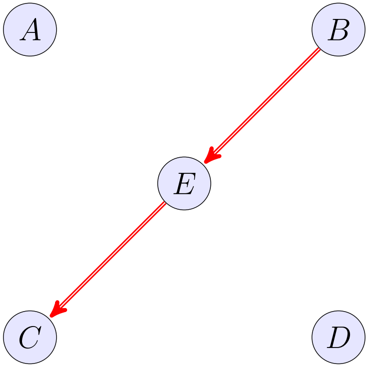
A possible temporal evolution...

1. $A \rightarrow E \rightarrow C$



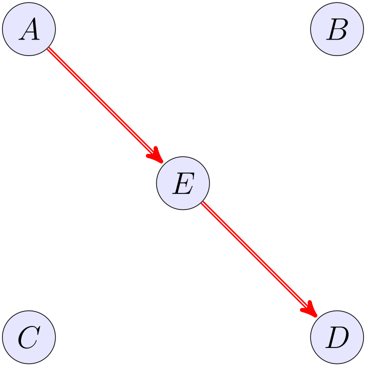
A possible temporal evolution...

1. $A \to E \to C$
2. $B \to E \to C$



A possible temporal evolution...

1. $A \to E \to C$
2. $B \to E \to C$
3. $A \to E \to D$



A possible temporal evolution...

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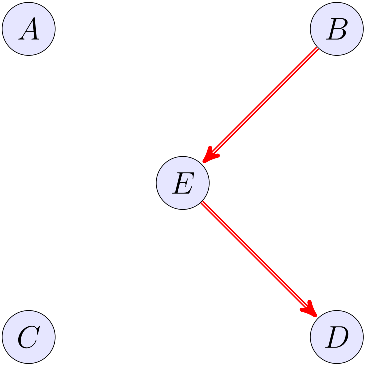
3. $A \to E \to D$
4. $B \to E \to D$



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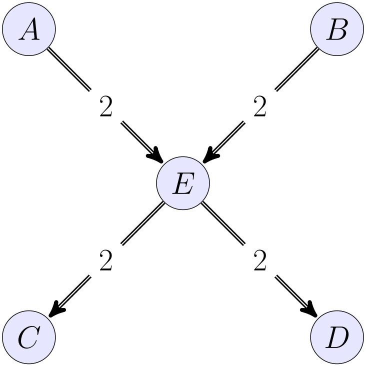
Well mixed!



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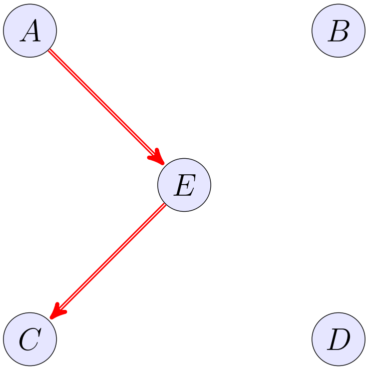

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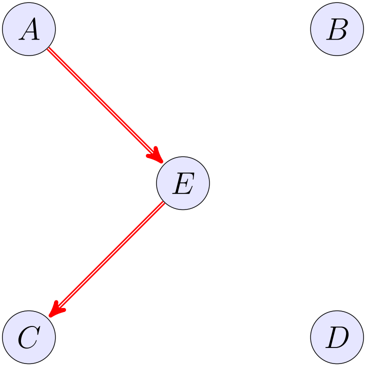

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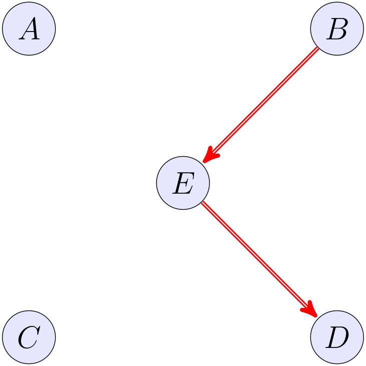

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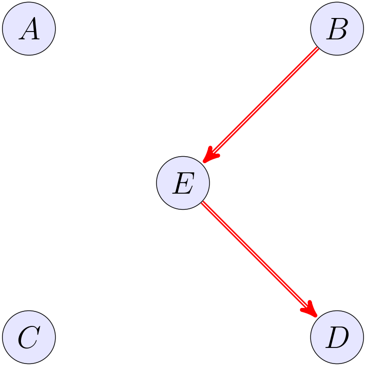

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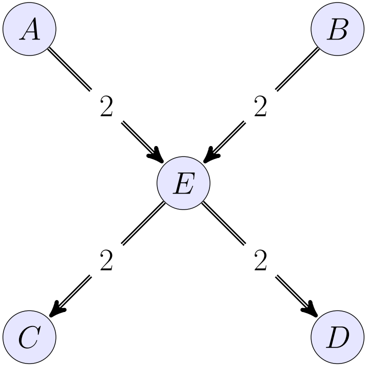
4. $B \to E \to D$

node E has a preference to mediate nodes A,C and B,D. $\to$ It has Betweenness Preference!

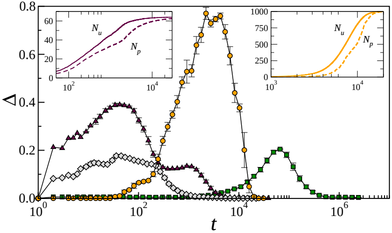


Influence on dynamical processes

Influence on dynamical processes

Susceptible-Infected epidemic model

$$\Delta = (N_u(t) - N_p(t))/N_u(t)$$

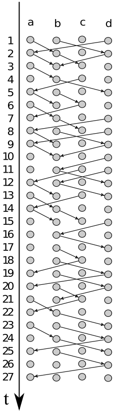


The uncorrelated model significantly overestimates the average number of infected individuals! (even up to 80%)

[Pfitzner, Scholtes, Garas, Tessone, Schweitzer, *Phys. Rev. Lett.* (2013)]

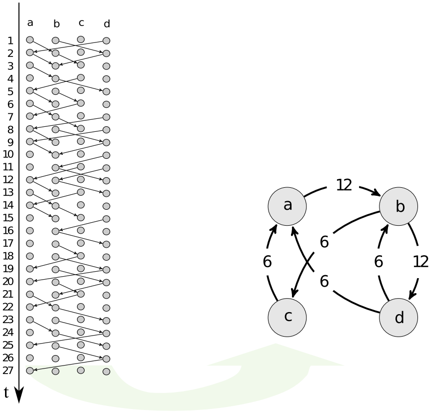
Definitions...

- Temporal network: $G^T = (V, E^T)$, $a, b \in V$, and $(a, b; t) \in E^T$.

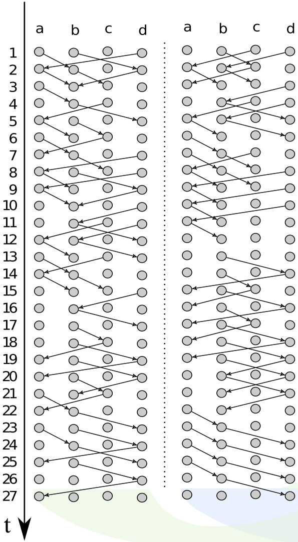


Definitions...

- Temporal network: $G^T = (V, E^T)$, $a, b \in V$, and $(a, b; t) \in E^T$.
- First order representation: $G^{(1)} = (V, E^{(1)})$, $E^{(1)} \subseteq V \times V$.
- Weight function $w_{ij}^{(1)}$: the relative number of edge occurrences.



But...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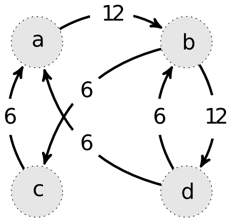
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State space extension

2^{nd}-order time-aggregated net.: $G^{(2)} = (V^{(2)}, E^{(2)})$, $V^{(2)} = E^{(1)}$

- nodes $e \in V^{(2)}$ represent edges in G^T.
- edges $(e_1, e_2) \in E^{(2)}$ represent time-respecting paths of length 2.
- weights $w^{(2)} : E^{(2)} \to \mathbb{R}$ capture the statistics of two-paths in G^T.

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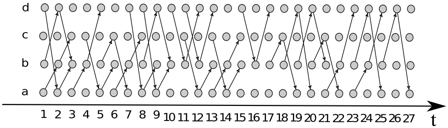

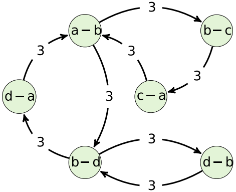

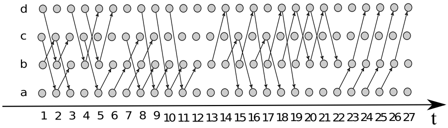

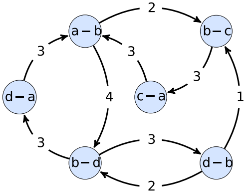

The transition matrix $\mathbf{T}^{(2)}$

- Using $w^{(2)}$, for $e_1, e_2 \in V^{(2)}$ we define the entries $T^{(2)}_{e_1,e_2}$ of a row stochastic matrix $\mathbf{T}^{(2)}$:

$$T^{(2)}_{e_1 e_2} := w^{(2)}\left(e_1, e_2\right)\left(\sum_{e' \in V^{(2)}} w^{(2)}\left(e_1, e'\right)\right)^{-1}$$

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- $\mathbf{T}^{(2)}$ is a transition matrix for a random walker in $G^{(2)}$.
- we obtain a second-order Markov model generating contact sequences which preserve:
  - the weights in the first-order time-aggregated network
  - the topology of the temporal network underlying $G^{(2)}$

The slow-down/speed-up factor

- Laplacian dynamics of G^T related to spectral properties of $\mathbf{T}^{(2)}$
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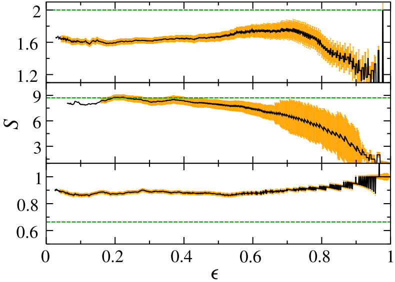
- For the random walk convergence due to non-Markovian properties of a G^T, $\mathcal{S}^*(\mathbf{T}^{(2)})$ provides:
  - the upper bound of the slow-down
  - the lower bound of the speed-up

Slow-down and speed-up



Speed-up/slow down factor for different datasets, as a function of the distance to the stationary distribution

- For the e-MID data (2012, all data), $\mathcal{S}^*(\mathbf{T}^{(2)}) \cong 1.90$
- The spreading processes are much slower that an aggregated representation indicates

[Scholtes, Wider, Pfitzner, Garas, Tessone, Schweitzer, *arXiv:1307.4030* (2014)]

Conclusions: Topological properties of temporal networks

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- State space expansion
- First step. Longer expansions needed when non-Markovian properties are more important